Light-Quark Mesons and Four-Quark Condensates at Finite Temperature

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Abstract

We propose an analog of the familiar gap equation for the case of four-quark condensates at finite temperature. The condensates of interest correspond to scalar, vector, psudoscalar, axial vector, and tensor Dirac structures. Working with correlators at zero chemical potential without factorization, we arrive at coupled equations for these four-quark condensates and the masses of certain light-quark mesons. We study the temperature dependence of the four-quark condensates and masses; in one of our models, factorization of the four-quark condensates is shown to be increasingly violated as the temperature is increased toward T_c. The 2^{++} tensor mesons $a_2(1320)$-$f_2(1270)$ are identified as especially sensitive probes of the four-quark condensates.

1 Introduction

The early universe is believed to have evolved through a stage beginning about 100 ps and ending about one μ sec after the big bang, during which time leptons had acquired their masses and strongly interacting matter existed primarily as a plasma of quarks and gluons. As the universe further expanded and cooled, when the temperature reached the value $T = T_c \simeq 200$ MeV matter began to undergo a phase transition as quark condensates and possibly hadrons began to form. Upon further expansion and cooling, this transition ran to completion with all quarks and gluons then having combined into hadrons.

Our paper concerns the physics that developed as the universe passed from $T \simeq T_c$ to $T = 0$. The relevant issues are identified by focusing on mesons in such a medium. A large body of work on vector mesons at finite T already exists, having been motivated by the fact that vector mesons can couple directly to photons providing an experimental lepton-pair signal of the masses and widths of these particles. However, without the factorization assumptions upon which most of this work relies, and which we describe in detail below, all the mesons become coupled through four-quark condensates. In the present paper we describe this physics without factorization, deriving a set of coupled equations for the meson masses and for the relavant four-quark condensates, and giving solutions with various scenarios for the finite-T spectral densities and two- and four-quark condensates.

The quantitative characterization of this sequence of events lies beyond the capabilities of theoretical analysis at present. However, because of its high interest, experimental programs have been planned to create similar conditions in the laboratory. It is the explicit aim for the RHIC (Relativistic Heavy Ion Collider) project at Brookhaven National Laboratory (BNL) to initiate a transition to matter at $T \approx 200$ MeV or greater, the quark-gluon plasma, within a large enough volume, and for a sufficiently extended time, to allow the study of its properties. This is envisioned to occur when the matter of large nuclei is compressed and heated as a result of violent,

head-on collisions between nuclei that have been accelerated to relativistic energies and allowed to collide in the RHIC storage ring. Encouraging evidence from preliminary experiments at the AGS at BNL and the SPS at CERN that a hot, dense fireball is formed in relativistic heavy ion collisions [1] keeps interest high in the RHIC program. The experimental determination of the nature of this transition is expected to guide the development of theoretical methods to treat the same physics.

In this paper we will develop analytical tools capable of addressing the temperature dependence of the four-quark condensates characterizing the quark-gluon plasma that may be created in this fashion. We are particularly interested in the four-quark condensates because they are fundamental quantities that characterize the nonperturbative vacuum in quantum chromodynamics (QCD) and are needed within the framework of QCD sum rules [2] to describe properties of both mesons (for a review, see Ref. [3]) and baryons [4]. They are especially important in the case of the light-quark mesons, which are the lowest excitations of the physical vacuum. The seminal work on this problem at $T \neq 0$ was done by Bochkarev and Shaposhnikov, Ref. [5]. They investigated an important four-quark condensate, namely that contributing to the mass of the ρ meson, using QCD sum rules extended to finite temperature.

Bochkarev and Shaposhnikov introduced a dynamical model for the temperature dependence of this four-quark condensate, thus avoiding the so-called factorization approximation, according to which each four-quark condensate becomes trivially proportional to the square of the familiar quark condensate, assuming saturation by the vacuum. A number of other authors have also calculated the mass of the rho meson using QCD sum rules at finite T [6, 7], generally using factorization. For the related physics of hadrons at $T = 0$ but finite density it has also been seen [8] that the factorization of the four-quark condensates is a major question, and that, e.g., in nuclear matter the theoretical result for the nucleon mass is criticaly dependent on the factorizaton assumption. In a study of the $\Delta(1236)$ in nuclear matter it has been shown that factorization of four-quark condensates is not valid for hadrons in a medium of non-zero density [9]. At zero baryon density there is evidence that it is a bad approximation [10]; however, because it is simple and convenient the approximation is widely

used. To our knowledge, the present work is the first attention given to the study of four-quark condensate in dynamical models since the original work of Bochkarev and Shaoposhnikov.

An important set of four-quark condensates are those of the form

$$\hat{Q}^{\Gamma} \equiv \langle 0| : \bar{q}\Gamma t^a q \bar{q}\Gamma t^a q : |0\rangle \tag{1}$$

where t^a are the SU(3) Gell-Mann color matrices and Γ corresponds to the set of five independent Dirac structures $\Gamma = 1$, γ_5, γ_μ,$\gamma_5\gamma_\mu$, and $\sigma_{\mu\nu}$, with an implied contraction over the Dirac indices. It is particularly relevant for the present work that these condensates make major contributions to the masses of the light-quark mesons generated by the corresponding five quark currents,

$$J^{\Gamma}(x) \quad = \quad \bar{q}(x)\Gamma q(x), \tag{2}$$

where q(x) are the u,d-quark fields contracted over their color labels. The mesons generated by a particular current $J^{\Gamma}(x)$ are those that are excited from the vacuum by the action of the current, i.e., those that satisfy

$$< 0|J^{\Gamma}(0)|meson(\Gamma) > \quad = \quad g_{\Gamma}\Psi_{\Gamma}(\vec{p}) \neq 0, \tag{3}$$

where $\Psi_{\Gamma}(\vec{p}, \lambda)$ is a wave function characteristic of the meson and g_{Γ} is a structure parameter. We will often refer to the mesons and other quantities associated with the five different Dirac Γ by *s, ps, v, av*, and t_σ, respectively.

In our discussion of the mesons, we will also be interested in the axial, tensor and axial tensor mesons because their masses depend on the same set of five four-quark condensates. The currents required for these mesons [3] are somewhat more complicated than the forms given in Eq. (2). One should also note that to determine the mass in the scalar (0^{++}) case, the current must include scalar glueball mixing; this is discussed in detail below. The most important mesons associated with each current will be specified below.

2 QCD Sum Rules for Mesons

The use of QCD sum rules for our purpose is suggested because this framework makes a relatively simple and explicit connection between the four-quark condensates and light meson masses. QCD sum rules have a number of drawbacks, however, such as uncertainty in how to treat the continuum, which becomes more and more serious issue as the temperature is raised. For this and other reasons, we do not aim to make predictions of QCD but rather develop QCD sum-rule based models, in which parameters are introduced to account for the temperature dependence of the continuum. We find our results reasonable, and we believe our *ad hoc* assumptions are capable of being improved as experimental results warrent it.

2.1 General Considerations

In the method of QCD Sum Rules [2], the propagator of a hadron is represented by a two-point function, usually called a correlator, defined as

$$\Pi^\Gamma(p) \quad = \quad i\int d^4x e^{ix\cdot p} < 0|T[J^\Gamma(x)J^\Gamma(0)]|0 >, \tag{4}$$

where for light-quark mesons the currents $J^\Gamma(x)$ are given in Eq. (2) and other forms specified below. This differs from the two-point function of standard field theories in that the $J^\Gamma(x)$'s are composite. Similar correlators are introduced in the study of baryons, but in the present work we focus only on the mesons for reasons discussed above.

A QCD sum rule for a quantity of interest is possible when the correlator is saturated by hadronic states, i.e. by using Eq. (3) in Eq. (4), producing a dispersion relation in which the quantity of interest appears explicitly. The expression resulting from this method of evaluation is often called the right-hand side (R.H.S.) of the sum rule. One form for the dispersion relation, used in the present work, is:

$$\Pi^\Gamma_{R.H.S.}(p) = \int_{-\infty}^{\infty} \frac{\rho(u,\vec{p})du}{u - p_o + i\eta} \tag{5}$$

with

$$\rho(E,\vec{p}) = \sum_{\lambda\alpha}\int \frac{d^3\vec{p}_\alpha}{(2\pi)^3}\langle 0|J^\Gamma|m_\alpha(\vec{p},\lambda)\rangle\langle m_\alpha(\vec{p},\lambda)|J^\Gamma|0\rangle\frac{(2\pi)^3\delta(\vec{p}-\vec{p}_\alpha)}{2E_\alpha}\delta(E-E_\alpha), \tag{6}$$

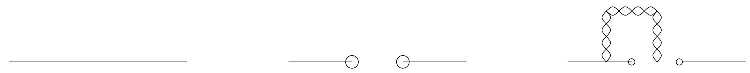

Figure 1: Quark propagator.

where $|m_\alpha(\vec{p},\alpha)\rangle$ of 4-momentum $(E_\alpha,\vec{p})$ is a complete set of intermediate states of quantum numbers λ, including the continuum.

To make a connection to QCD, the correlator is then evaluated microscopically by expanding in operators of increasing dimension using an operator product expansion (o.p.e). The resulting evaluation is often called the left-hand side (L.H.S.). For the quark propagator in space-time,

$$\begin{aligned} S_q(x) &= <0|T[q(x)\bar{q}(0)]|0>, \\ &= S_q^{P.T.}(x) + S_q^{N.P.}(x), \end{aligned} \tag{7}$$

with

$$S_q^{N.P.}(x) = <0| : q(x)\bar{q}(0) : |0>, \tag{8}$$

where $S_q^{P.T.}(x)$ is the usual perturbative quark propagator, illustrated in Fig. 1a, and $S_q^{N.P.}(x)$ is the nonperturbative quark propagator, which vanishes in the perturbative vacuum. The operator product expansion gives

$$S_q^{N.P.}(x) = \frac{-1}{12} <0| : \bar{q}q : |0> + \frac{x^2}{3 \cdot 2^6} <0| : \bar{q}\sigma \cdot Gq : |0> + ... \tag{9}$$

in terms of the quark condensate, whose value is given by

$$\langle 0| : \bar{q}q : |0\rangle = -(0.25 \text{ GeV})^3, \tag{10}$$

a mixed condensate, and so forth as illustrated in Figs. 1b, 1c. The justification for the o.p.e. is that the evaluation of $\Pi^\Gamma(p)$ is done using a Borel transformation with the Borel mass in the range of about 1 GeV, allowing a short-distance expansion.

The microscopic derivation of the correlator $\Pi^\Gamma(p)$ is completed by using the expansion of S_q(x) given in Eqs. (7,9), leading to the perturbative term illustrated in

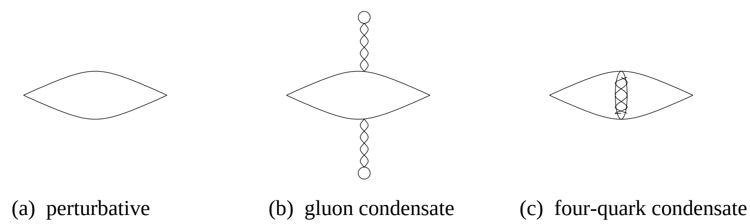

Figure 2: O.P.E. for meson correlator.

Fig. 2a. To this is added terms with gluons attached to two or more quark propagators, with the dimension four gluon condensate term illustrated in Fig. 2b. Finally, there are important contributions that come from the four-quark terms illustrated in Fig. 2c and that are the focus of this work. For light-quark mesons, the quark condensate terms are suppressed because they are multiplied by a small number, the mass of an u or d quark. Consequently, the most important nonperturbative contributions to the light-meson masses are four-quark condensates Fig. 2c, and the gluon condensate, Fig. 2b.

We will denote the four-quark condensates corresponding to the correlator Π^Γ as $\hat{\hat{Q}}^\Gamma$. These have the following form: $\hat{\hat{Q}}^v \equiv \hat{Q}^{av} + \frac{2}{9}|J_0|^2$, $\hat{\hat{Q}}^{av} \equiv \hat{Q}^v + \frac{2}{9}|J_0|^2$, $\hat{\hat{Q}}^s \equiv \hat{Q}^{t\sigma} + \frac{2}{3}|J_0|^2$, $\hat{\hat{Q}}^{t\sigma} \equiv \hat{Q}^s + \frac{1}{36}|J_0|^2$ and $\hat{\hat{Q}}^{ps} \equiv \hat{Q}^{at\sigma} + \frac{2}{3}|J_0|^2$, where $\hat{Q}^\Gamma$ is defined in Eq. (1) and

$$|J_0|^2 \equiv \langle 0| : \bar{q}\gamma_\mu t^a q \sum_f \bar{q}_f \gamma^\mu t^a q_f : |0\rangle. \tag{11}$$

As the four-quark condensates are mediated by gluon exchange, they contribute to the correlator proportional to $\alpha_s = g_s^2/4\pi \approx 0.7$ at a scale of $\mu = 200$ MeV [2], where g_s is the strength of the quark-gluon coupling. Note that we have given four-quark condensates corresponding to correlators for all five Dirac structures, including $\Pi^{t\sigma}$, and that at_σ refers to the current J^Γ with $\Gamma = \gamma_5\sigma_{\mu\nu}$. Other four-quark condensates are needed for the correlators of the axial, tensor and axial tensor mesons, and these are defined below.

As we have noted, the square of the quark condensates instead of $\hat{\hat{Q}}^\Gamma$ appears in versions of the theory when the *factorization* approximation is applied to the four-quark condensates. This approximation has been successful in describing the properties of

hadrons at $T = 0$ in free space (see, however, Ref. [10]). Since factorization may not be valid when $T \neq 0$, in developing our ideas we will try to avoid it as far as possible. Consequently, we will generally be using the exact representation of the four-quark condensates as given in Fig. 2c. At $T = 0$, the gluon condensate has the familiar value

$$g_s^2\langle 0| : G^a_{\mu\nu}G^{a\mu\nu} : |0\rangle \equiv \langle g_s^2 G^2\rangle \simeq 0.47 \text{ GeV}^4. \tag{12}$$

2.2 Extension to Finite Temperature

We want to investigate the excitation modes of hadronic matter brought into contact with a heat bath. Since we assume temperature equilibrium, the heat bath is the local environment for describing the properties of mesons propagating in the early universe or in the quark-gluon plasma created in the aftermath of colliding relativistic heavy ions in RHIC experiments. The heat bath elevates the temperature of all matter in the vacinity of the meson in question to temperature T, including the constituents of the vacuum.

Statistical mechanics allows us to calculate the thermal average $\Pi^\Gamma(p,T)$,

$$\Pi^\Gamma(p,T) \;\; = \;\; i\int d^4x e^{ix\cdot p}\theta(x_0)\langle\langle 0|J^\Gamma(x)J^\Gamma(0)]|0\rangle\rangle, \tag{13}$$

where $p = (\omega, \mathbf{p})$ and where $\langle\langle ...\rangle\rangle$ stands for the Gibbs average. As in the case of the corresponding operator given in Eq. (4), the poles of this operator correspond to the excitation spectrum of the medium. Note that at finite temperature, one works with the retarded rather than the causal propagator.

The quantity in Eq. (13) may be evaluated in field theory using the Matsubara formalism [11], which recognizes the similarity between the time evolution operator introduced in field theory at $T = 0$ and the Grand canonical ensemble introduced at $T \neq 0$. The relevant formal distinction is the replacement of *time* $\rightarrow i/T$ to obtain the $T \neq 0$ results from the $T = 0$ results. Since the limits of integration are finite and bounded by T at $T \neq 0$, a boundary condition must be specified that is not needed in the case of the case of field theory at zero temperature. These differences translate into

the following rule for transforming a Feynman diagram at $T = 0$ to a corresponding diagram at finite temperature: the momentum $p^\mu \to (i\omega_n, \mathbf{p})$, where

$$\begin{array}{lcll} \omega_n & \to & 2\pi T n & \text{bosons (gluons), and} \\ \omega_n & \to & 2\pi T(n+\frac{1}{2}) & \text{fermions (quarks),} \end{array} \tag{14}$$

where n runs over positive and negative integers (including zero). Thus, instead of integrals over ω in Feynman diagrams, one has sums over n. When Feynman diagrams of a causal Green's function are translated in such a fashion, it can be shown that one recovers the Fourier transform of the advanced (retarded) real-time Green's function (at finite temperature T) when the analytical continuation $\omega_n \to i\omega + (-)\eta$ is made [12].

The Matsubara formalism may be applied to the correlator in Eq. (4) and hence to calculate the meson masses as a function of temperature using QCD sum rules. One merely has to observe that the o.p.e. is made using Euclidian variables, so the appropriate replacement of the Matsubara frequency is now $\omega_n \to \omega$. We will be interested in the excitation spectrum at $\mathbf{p}$=0, so we will then be making the Borel transform (see Sect. 2.3) on the variable ω.

In obtaining the QCD sum rule, one must recognize that the on the right-hand side, the residue at the pole of the advanced (or retarded) propagator is the familiar one except for the presence of the occupation probability factor (for bosons) $\coth(m_\Gamma/2T)$. On the left-hand side, the perturbative quark term Fig. 2a is evaluated in accord with the above considerations in terms of the Matsubara propagator

$$\frac{1}{i\omega_n\gamma_0 - \gamma\cdot\mathbf{p} - m}, \tag{15}$$

and the nonperturbative terms are evaluated with the temperature-dependent condensates.

The temperature dependence of the quark condensate $\langle\langle \bar{\mathrm{q}}\mathrm{q}\rangle\rangle$ has been evaluated in lattice gauge theory [13] and in the model of Nambu and Jona-Lasinio [14] extended to finite temperature using the Matsubara formalism. In the latter picture, the two-quark condensate drops toward zero at a temperature of about 160 MeV at zero chemical potential. The models of the temperature dependence of the ρ meson mass developed in

Refs. [6, 7] were developed using factorized four-quark condensates with $\langle\langle\bar{q}q\rangle\rangle$ having a temperature dependence of the general character found there. The temperature dependence or the quark condensate has also be studied in instanton models [15].

As we have remarked, if one avoids the factorization approximation then the actual contribution of $\langle\langle\bar{q}q\rangle\rangle$ to the light-quark mesons is small and may be ignored. One must instead use the four-quark condensates directly; this is the proceedure we will follow. As the temperature dependence of these is not generally known, we will of course be using the temperature dependence as it comes from our theory.

The remaining non-perturbative quantity that needs to be specified is the gluon condensate. At finite temperature, we may write for the gluon condensate

$$\langle G^2\rangle \simeq \langle : G^2 :\rangle + \langle G^2\rangle_T^{P.T.} \tag{16}$$

where the perturbative term $\langle G^2\rangle_T^{P.T.}$ has been calculated by Kapusta [16] and found to be

$$\langle G^2\rangle_T^{P.T.} \simeq \frac{8\pi^2}{15} T^5, \tag{17}$$

which at $T_c \simeq 200$ MeV is a very small fraction of the zero-temperature nonperturbative term, $\langle G^2\rangle_{T_c}^{P.T.} < \frac{1}{10}\langle : G^2 :\rangle$. This leads to our assumption, also made in Ref. [6, 7], that the temperature dependence of $\langle : G^2 :\rangle_T$ is weak. In the present work we assume that

$$\langle G^2\rangle_T \simeq \langle G^2\rangle. \tag{18}$$

This also means that the gluon condensate is nonvanishing for temperatures $T > T_c$, so that the vacuum is still nonperturbative.

2.3 QCD Sum Rules for Meson Masses

In this subsection, we present the results for the QCD sum rules for meson masses [3] extended to finite temperature using the Matsubara formalism [11]. In the next subsection we will introduce similar sum rules, with however some important differences, for the four-quark condensates.

The light-quark mesons that have been succesfully treated in QCD sum rules include the lightest that couple to a given J^Γ as specified in Eq. (2). Generally, these mesons come in nearly degenerate isospin doublets. For the light-quark mesons corresponding to the elementary Dirac operators, we have the following mesons to consider: the $\mathrm{f}_0(1370)$-$\mathrm{a}_0(1450)$ [17] in the 0^{++} scalar channel (Γ=1); the $\rho(776)$-$\omega(783)$ [2] in the 1^{--} vector channel ($\Gamma = \gamma_\mu$); $\mathrm{a}_1(1260)$-$\mathrm{f}_1(1285)$ [18] in the 1^{++} axial vector channel ($\Gamma = \gamma_5\gamma_\mu$). The $\pi(140)$-$\eta(549)$ in the 0^{-+} psudoscalar channel ($\Gamma = \gamma_5$), is an exception for which QCD sum rules are no good. In the two scalar channels, the f_0 and a_0 have a different character in that only the f_0 couples directly to the 0^{++} scalar glueball, but this does not affect our calculation of the $\bar{q}q$ part of the correlator.

The correlators $\Pi^\Gamma(p)$ generally have a rich tensoral structure, and it is sufficient for our purposes to consider only one of these, which we will refer to as Π'^Γ. In the case of the vector, axial vector, and scalar mesons Π'^Γ is related to Π^Γ of Eq. (4) respectively by $\Pi'^{v_\parallel} = \pi p^{-2} g^{\mu 0} g^{\nu 0} \Pi^v_{\mu\nu}$ for the longitudinal correlator of the $\rho(776)$-$\omega(783)$; $\Pi'^{av} = -\frac{\pi}{3} g^{\mu\nu} \Pi^{av}_{\mu\nu}$ for the $\mathrm{a}_1(1260)$-$\mathrm{f}_1(1285)$; and $\Pi'^s = \pi \Pi^s$ for the $\mathrm{f}_0(1370)$-$\mathrm{a}_0(1450)$. Noting the near-degeneracy of the I=0,1 mesons of each type except the ps, one can simplify the theory considerably by assuming an exact isospin degeneracy; in this case the correlators are multiplied by a factor of two relative to the value in the absence of isospin symmetry [3].

We next characterize the correlators as they arise from the quark current, coming back to the glueball contribution of the 0^{++} mesons later. A convenient summary of the various ingredients may be found in the review [3], except for the extension to finite temperature, which follows the arguments found above.

The L.H.S. of the correlators arising from the quark current have the following form in momentum for the s, v, and av cases,

$$\Pi'^\Gamma_{L.H.S}(Q^2, T) = a_\Gamma I_n(-p^2) + b_\Gamma \frac{\langle \frac{\alpha}{\pi} G^2 \rangle}{(-p^2)^{n(\Gamma,b)}} + c_\Gamma \frac{\hat{\hat{Q}}^\Gamma}{(-p^2)^{n(\Gamma,c)}}. \tag{19}$$

In these expressions, $-p^2 = Q^2$ is the Euclidian momentum, and the function $I_n(-p^2)$ characterizes the perturbative quark terms (see Fig. 2(a)) and has the following form

$$I_n(-p^2) \equiv i_n(-p^2) \tag{20}$$

for the s and av cases, and

$$I_n(-p^2) \equiv i_0(-p^2) \quad + \quad \frac{2}{-p^2}\int_0^\infty d\omega^2 n_F(\frac{\omega}{2T}) \\ -\frac{2}{3(-p^2)}\int_{4m_\pi^2}^\infty d\omega^2 n_B(\frac{\omega}{2T}). \tag{21}$$

for the $v_\parallel$ case. Here $n_B(z) = (e^z - 1)^{-1}$, $n_F(z) = (e^a + 1)^{-1}$, and

$$i_n(-p^2) \equiv \frac{1}{n!}\int_0^{S_0} d\omega^2 \frac{\omega^{2n}}{-p^2+\omega^2}\tanh(\omega/4T). \tag{22}$$

The last integral in Eq. (21) describes the damping due to thermal excitation of pions [5] and is particular to the longitudinal channel. We have assumed that the contribution of the continuum to the dispersion relation begins at $s = S_0$, and that this piece of the R.H.S. is transferred to the L.H.S. of the sum rule, where it regulate the large-s contributions of the perturbative terms. For this reason, the integrals in Eq. (22) run only up to S_0. This corresponds to the usual treatment of the continuum at $T = 0$.

The following light-quark mesons, which have slightly more complicated currents, have also been successfully treated by QCD sum rules: The $\mathrm{D}_1(1235)$ [18] in the axial 1^{+-} channel ($J_\mu^{(1^{+-})} = i\bar{q}\gamma_5 \overleftrightarrow{\partial}_\mu q$); the $\mathrm{a}_2(1320)$-$\mathrm{f}_2(1270)$ [19] in the tensor 2^{++} channel ($J_{\mu\nu}^{(2^{++})} = i\bar{q}(\gamma_\mu \overleftrightarrow{\partial}_\nu + \gamma_\nu \overleftrightarrow{\partial}_\mu)q$); and the $\mathrm{H}_2(1670)$ [19] in the axial tensor 2^{-+} channel ($J_{\mu\nu}^{(2^{-+})} = i\bar{q}(\gamma_\mu\gamma_5 \overleftrightarrow{\partial}_\nu + \gamma_\nu\gamma_5 \overleftrightarrow{\partial}_\mu)q$.

The correlator Π'^Γ for the axial meson has the same form as given in Eq. (19). For the tensor and axial tensor mesons, Π'^Γ has the form

$$\Pi'^\Gamma_{L.H.S}(Q^2,T) = a_\Gamma I_n(-p^2) + b_\Gamma(-\log(-p^2))\langle\frac{\alpha}{\pi}G^2\rangle + c_\Gamma\frac{\hat{\hat{Q}}^\Gamma}{(-p^2)^{n(\Gamma,c)}}. \tag{23}$$

The following three $\hat{\hat{Q}}$ correspond to the axial (1^{+-}), tensor (2^{++}), and axial tensor (2^{-+}) mesons respectively: $\hat{\hat{Q}}^a = \hat{Q}^{ps} - \frac{1}{9}|J_0|^2$, $\hat{\hat{Q}}^t = \hat{Q}^v$, and $\hat{\hat{Q}}^{at} = \hat{Q}^{av}$, where we have introduced rather obvious abbreviations for these channels. The appropriate $I_n(-p^2)$ is that of Eq. (20) in these cases.

The coefficients of Eqs. (19,23) are given in Table 1. We refer the reader to the original literature [2, 3, 18, 17] for the details not given here.

To obtain the sum rule, we make the Borel transform

$$\Pi'^\Gamma(M_B^2,T) \equiv \hat{L}_{M_B}\Pi'^\Gamma(Q^2,T) = \lim_{Q^2,n\to\infty,Q^2/n=M_B^2}\frac{(Q^2)^{n+1}}{n!}(\frac{d}{d(Q^2)})^2\Pi'^\Gamma(Q^2,T)) \tag{24}$$

of both the R.H.S. and left-hand side. After making the Borel transform of the meson-mass sum rules, we may write the L.H.S. of the sum rule in the following generic fashion

$$\Pi'^{\Gamma}_{L.H.S}(M_B^2,T) = A_\Gamma M_B^{2(n+1)} e_n(M_B) + B_\Gamma \frac{\langle \frac{\alpha}{\pi} G^2 \rangle}{M_B^{2n(\Gamma,B)}} + C_\Gamma \frac{\hat{\hat{Q}}^{\Gamma}}{M_B^{2n(\Gamma,C)}}, \tag{25}$$

The function $e_n(M_B)$ is given in all cases by $M_B^{2(n+1)} e_n(M_B) \equiv \hat{L}_{M_B} I_n(Q^2)$, with

$$\hat{L}_{M_B} i_n(Q^2) \equiv \frac{1}{n!} \int_0^{S_0} d\omega^2 \omega^{2n} e^{-\frac{\omega^2}{M_B^2}} \tanh(\frac{\omega}{4T}). \tag{26}$$

Note, in the case of the longitudinal channel [5] for the vector meson, we have from Eqs. (20) and (21)

$$M_B^2 e_0(M_B) \equiv \int_0^{S_0} d\omega^2 e^{-\frac{\omega^2}{M_B^2}} \tanh(\frac{\omega}{4T}) + 2\int_0^{\infty} d\omega^2 n_F(\frac{\omega}{2T}) - \frac{2}{3} \int_{4m_\pi^2}^{\infty} d\omega^2 n_B(\frac{\omega}{2T}). \tag{27}$$

The coefficients A, B, C, and the various n's, are given in Table 2 for all the cases.

In the case of the 0^{++} meson, we follow the philosophy of Ref. [17] in which the scalar meson is treated as an hybird. i.e., the current for the 0^{++} case is

$$J^{0^{++}} = \beta J^s + (1 - |\beta|) J^{GB(0^{++})}, \tag{28}$$

where J^s is the scalar quark current defined in Eq. (2) and $J^{GB(0^{++})}$ is the scalar glueball current. Details are given in Ref. [17]. Our approach differs from that of Ref. [3, 18] because the mesons considered there correspond to the $\mathrm{f}_0(980)$ and $\mathrm{a}_0(980)$ mesons, which are not $\bar{q}q$ mesons. Instead, we consider the 0^{++} to be the $\mathrm{f}_0(1370)$ and $\mathrm{a}_0(1450)$ mesons. Because these mesons are hybrid, there is in addition to the quark current and the correlator arising from the coupling of the $\bar{q}q$ pair to the glueball, giving rise to a "glueball" contribution

$$\Pi'^{Glueball}_{L.H.S} = 4\pi M_B^6 (\frac{\alpha_S}{\pi})^2 + 8\pi^3 (\frac{\alpha_S}{\pi})^2 \Gamma^{(6)} + (\frac{\alpha_S}{\pi})^3 \frac{8\pi^5 \Gamma^{(8)}}{M_B^2}. \tag{29}$$

To obtain the full correlator for the scalar meson, we combine this with the quark contribution in Table 2,

$$\Pi'^{0^{++}}_{L.H.S} = \beta^2 \Pi'^{quark,0^{++}}_{L.H.S} + (1 - |\beta|)^2 \Pi'^{glueball}_{L.H.S.}, \tag{30}$$

where the value $\beta = 0.7$ reproduces the mass of the $M^{0^{++}}$=1370 MeV. Note that the cross term, proportional to $\beta(1-|\beta|)$, does not appear in the expression in Eq. (30) because this term vanishes under the Borel transform in the form we use.

Our correlator does not agree exactly with the one found in Ref. [17]. As in the case of the other mesons, we multiply the quark term by a factor of two to take into account that we are mixing taking the isoscalar and isovector mesons to be degenerate. Additionally, we use an *unsubtracted* dispersion relation and regulate the strong dependence on M_B in the glueball similar to the regulation of the continuum for quarks. Since the glueball makes a relatively small contribution for this meson, these differences are relatively unimportant numerically.

To complete the sum rule for the meson masses, we need to specify the expression for the phenomenological correlator, $\Pi'^{\Gamma}_{R.H.S.}$. In all cases, $\Pi'^{\Gamma}_{R.H.S.}(M_B^2, T)$ is proportional to $e^{-m_\Gamma^2/M_B^2}$, where m_Γ is the mass of the meson created by current J^Γ. The meson-mass sum rule is obtained by equating the the logarithmic derivitive of the R.H.S. and L.H.S.,

$$\frac{\partial_{M_B^{-2}} \Pi'^{\Gamma}_{R.H.S.}}{\Pi'^{\Gamma}_{R.H.S.}}(M_B^2, T) = \frac{\partial_{M_B^{-2}} \Pi'^{\Gamma}_{L.H.S.}}{\Pi'^{\Gamma}_{L.H.S.}}(M_B^2, T). \tag{31}$$

According to the theory, the sum rule is valid only in a region there the combination in Eq. (31) is relatively insensitive to M_B, which in practice means within a region where the sum rule has a plateau. Note that the logarithmic derivitive of the R.H.S. is just the square of the meson mass, and for this reason it is not necessary to know the constant of proportionality in $\Pi'^{\Gamma}_{R.H.S.}(M_B^2, T)$. Later, when we discuss the four-quark sum rules the constant of proportionality on the R.H.S. is quite important, and we will carefully discuss it for these cases.

We give some of the results for m_Γ in Table 3. We also give the values of the threshold parameters S_0^Γ and the factorized values of the four-quark condensates appropriate for $T = 0$. As T is increased, we cannot solve the QCD sum rules for meson masses without knowing the temperature dependence of the four-quark condensates. In Sect. 3 we present two models for this.

2.4 Four-Quark Condensate Sum Rules

We next want to look for a sum rule whose solution gives the four-quark condensates themselves. The correlators for the four-quark sum rules will be the correlators Π^{Γ}(p,T) in Eq. (13) for the currents J^{Γ} corresponding to the five elementary Dirac structures. The correlators are in most instances not the same as they were for the meson-mass sum rules. For example, the sum rules for Π^{t_σ} and Π^{ps} are going to be considered in this context. We will give our arguments in the next section why the use of the psudoscalar correlator the four-quark condensate may be justified while it is not justified for m_π, as noted earlier.

One can easily verify that J^{t_σ} couples to the 1^{--} channel from the fact that a nonvanishing antisymmetric tensor for the overlap in Eq. (3) can be constructed from the vector meson $|m(p,\lambda)\rangle$ intermediate state in $\Pi'^{t_\sigma}_{R.H.S.}$ as

$$\langle 0|\sigma_{\mu\nu}|m(p,\lambda)\rangle = g_{t_\sigma}(p_\mu \varepsilon_\nu^{(\lambda)}(p) - p_\nu \varepsilon_\mu^{(\lambda)}(p)), \tag{32}$$

where $\varepsilon_\mu^{(\lambda)}$ is the polarization vector for the vector meson. Because the current J^{t_σ} couples to the same 1^{--} channel as the ρ-ω, the t_σ channel is generally not considered as an independent case.

To identify the quantity that we will be solving the sum rules to obtain, consider first the R.H.S of $\Pi^{\Gamma}(p)$ for the case of $T=0$. In this case, we write for the residue at the pole in Eq. (6)

$$\sum_\lambda \langle 0|J^{\Gamma}|m_\alpha(\vec{p},\lambda)\rangle\langle m_\alpha(\vec{p},\lambda)|J^{\Gamma}|0\rangle = CF^{\Gamma}(\alpha)P_\alpha^{\Gamma}(\vec{p}) \tag{33}$$

where $P_\alpha^{\Gamma}(\vec{p}) = \sum_\lambda \Psi(\vec{p},\lambda)\Psi^\dagger(\vec{p},\lambda)$ is a projection operator whose form is specified by the quantum numbers for the particular meson in question, C is a overall scale common to all mesons Γ, and where $F^{\Gamma}(\alpha)$ is a function that characterizes the differences among the different mesons and whose form is not needed in this work. Explicit forms suggested for these functions may be found in Ref. [5, 20, 21, 22]. We give $P^{\Gamma}(p)$ along with the definitions of the correlators $\Pi'^{\Gamma}(Q^2,T)$ for each of the five elementary Dirac

operators in Table 4. Note that for the vector meson, we use the transverse correlator, $\Pi'^{v\perp} = -\pi g^{\mu\nu}\Pi^{v}_{\mu\nu}$.

We will be making some model assumptions about the spectrum of states $|m_\alpha\rangle$ that enter Eq. (33), one being to replace the continuum by a set of resonances having four-momentum $(|E_\alpha| \equiv \sqrt{\vec{p}^2 + m_\alpha^2}, \vec{p})$. In this case, consider the matrix element $\langle 0|J^\Gamma J^\Gamma|0\rangle$ and insert a complete set of states $|m_\alpha\rangle$ between the two currents to obtain

$$\begin{aligned}\langle 0|J^\Gamma J^\Gamma|0\rangle &= \sum_{\lambda\alpha}\int \frac{d^3\vec{p}_\alpha}{(2\pi)^3}\frac{1}{2E_\alpha}\langle 0|J^\Gamma|m_\alpha(\vec{p},\lambda)\rangle\langle m_\alpha(\vec{p},\lambda)|J^\Gamma|0\rangle \\ &= C\kappa^\Gamma \end{aligned} \tag{34}$$

where

$$\kappa^\Gamma \equiv \sum_\alpha F^\Gamma(\alpha)\int \frac{d^3\vec{p}_\alpha}{(2\pi)^3}\frac{\check{P}^\Gamma_\alpha(p)}{2E_\alpha}, \tag{35}$$

where $\check{P}^\Gamma_\alpha$ denotes contraction over the Dirac indices of P^Γ_α. The value of κ^Γ is dependent on various cutoff factors that are not known, so we will eventually fix the value of κ^Γ phenomenologically to assure that the four-quark condensates attain their factorized value for $T = 0$. We may solve Eq. (34) for the scale C to obtain

$$C = \langle 0|J^\Gamma J^\Gamma|0\rangle/\kappa^\Gamma. \tag{36}$$

Now, using Eqs. (6) and (36) we may write the sum in Eq. (33) as

$$\rho(E,\vec{p}) = \frac{\langle 0|J^\Gamma J^\Gamma|0\rangle}{\kappa^\Gamma}\int \frac{d^3\vec{p}_\alpha}{(2\pi)^3}\sum_\alpha F^\Gamma(\alpha)P^\Gamma_\alpha(\vec{p})\frac{(2\pi)^3\delta(\vec{p}-\vec{p}_\alpha)}{2E_\alpha}\delta(E-E_\alpha). \tag{37}$$

Note that with the help of Eq. (37) we are able to express the R.H.S. of the correlator $\Pi^\Gamma(p)$ in terms of the quantity $\langle 0|J^\Gamma J^\Gamma|0\rangle$ that is closely related to the four-quark condensate $\langle\langle 0|\mathrm{\bar{q}\Gamma t^a q \bar{q}\Gamma t^a q}|0\rangle\rangle$. Such an expression will enable us to find a set of sum rules for the four-quark condensates, independent of the QCD sum rules for meson masses.

The extension of Eq. (37) to the case of finite temperature is straightforward using the results of Sect. (2.2). We noted there that the R.H.S. for the case of finite temperature was similar to the R.H.S. at $T = 0$ except for the appearance of a temperature-dependent occupation probability. Since the quantity appearing on the R.H.S. is a

meson, the correct occupation probability is $\coth(\frac{E_\alpha}{2T})$. Then,

$$\rho_T(E,\vec{p}) = \frac{\langle\langle 0|J^\Gamma J^\Gamma|0\rangle\rangle}{\kappa^\Gamma(T)} \int \frac{d^3\vec{p}_\alpha}{(2\pi)^3} \sum_\alpha F^\Gamma(\alpha) P^\Gamma_\alpha(\vec{p}) \coth(\frac{E_\alpha}{2T}) \frac{(2\pi)^3\delta(\vec{p}-\vec{p}_\alpha)}{2E_\alpha} \delta(E-E_\alpha), \quad (38)$$

where

$$\kappa^\Gamma(T) \equiv \sum_\alpha F^\Gamma(\alpha) \int \frac{d^3\vec{p}_\alpha}{(2\pi)^3} \frac{1}{2E_\alpha} P^\Gamma_\alpha(p) \coth^2(\frac{E_\alpha}{2T}). \quad (39)$$

In the QCD sum rule for meson masses, the higher-lying states that contribute to the dispersion relation (i.e., the continuum, assumed to begin at $s = S_0$) are transferred to the L.H.S. of the sum rule, where they regulate the perturbative term at large momentum. We make the assumption that the continuum plays exactly the same role for the four-quark condensate sum rules, and therefore we characterize the continuum by exactly the same functions $I_n(M_B^2)$ that characterize the meson-mass sum rules. Here, we parametrize S_0 to be a function of temperature, the functional form carrying the burden of describing the dropping of the threshold S_0 as the temperature is raised, as well as the temperature dependence of $\kappa(T)$ in Eq. (39). With these asumptions, the R.H.S. of $\Pi^\Gamma(p)$ has the following form,

$$\Pi^\Gamma_T(\omega,\vec{p};T) = \frac{P^\Gamma(p)}{-p^2+m_\Gamma^2} \frac{\langle\langle 0|J^\Gamma J^\Gamma|0\rangle\rangle}{\kappa^\Gamma} \coth(m_\Gamma/2T). \quad (40)$$

The left-hand side of $\Pi^\Gamma(p,T)$ is expanded in an o.p.e. as before in the case of meson masses. The gluon condensate is treated as in the case of the meson-mass QCD sum rule. The correlator $\Pi'^\Gamma_{L.H.S.}(Q^2,T)$ has the same form as given in Eq. (19), with the treatment of the continuum as given in the preceeding paragraph. There is a glueball effect in the s channel now only in $\rho_T(E,\vec{p})$, since the current is purely of the form given in Eq. (2). The values of the coefficients are given in Table 5.

The temperature dependence of the four-quark condensate $\langle\langle 0|\bar{\mathrm{q}}\Gamma \mathrm{t^a q}\bar{\mathrm{q}}\Gamma \mathrm{t^a q}|0\rangle\rangle$ is treated as follows. We would like to relate it to the quantity $\langle\langle 0|J^\Gamma J^\Gamma|0\rangle\rangle$ that appears on the R.H.S. of the sum rule in order to obtain a set of equations to determine it. This connection is found by making a Fierz rearrangement of the quark operators appearing in $\langle\langle 0|\bar{\mathrm{q}}\Gamma \mathrm{t^a q}\bar{\mathrm{q}}\Gamma \mathrm{t^a q}|0\rangle\rangle$, so we write

$$\langle\langle 0|\bar{\mathrm{q}}\Gamma \mathrm{t^a q}\bar{\mathrm{q}}\Gamma \mathrm{t^a q}|0\rangle\rangle = \sum_{\Gamma'} \mathrm{c}_{\Gamma'}(\Gamma)\langle\langle 0|\mathrm{J}^{\Gamma'}\mathrm{J}^{\Gamma'}|0\rangle\rangle \quad (41)$$

making use of the following relationship among the Gell-Mann matrices

$$t^a_{\alpha\gamma} t^a_{\lambda\beta} = 2\delta_{\alpha\beta}\delta_{\gamma\lambda} - \frac{2}{3}\delta_{\alpha\gamma}\delta_{\lambda\beta}. \tag{42}$$

After making the Fierz rearrangement, the four-quark condensates $\hat{\hat{Q}}^{\Gamma}$ are related to $\hat{Q}^{\Gamma}$ as shown in Table 6. We have taken $|J_0|^2 = \hat{Q}^v$, which although not an exact result differs from it only by terms that vanish in the factorization approximation. This is the only place where we make a factorization assumption (other than at $T = 0$).

To obtain the four-quark sum rule, we make the Borel transform as given in Eq. (24) of both the R.H.S. and L.H.S. and then equate them,

$$\Pi'^{\Gamma}_{L.H.S.}(M_B^2, T) = \Pi'^{\Gamma}_{R.H.S.}(M_B^2, T). \tag{43}$$

As in the case of the meson-mass sum rule, the equality is possible only in a region where the sum rule is rather insensitive to M_B. The R.H.S. and L.H.S. of the four-quark sum rules are given in Tables 7 and 8, respectively. The L.H.S. correlators have the same form as given in Eq. (25). Note that these constitute five equations in the five unknowns $\langle\langle 0|\mathrm{\bar{q}\Gamma t^a q\bar{q}\Gamma t^a q}|0\rangle\rangle$ and other quantities. We will come back to the solution of this equation later in the next section.

3 Toward a Theory of Four-Quark Condensates at Finite Temperature

In this section, we will present two models of the four-quark condensates and the corresponding meson masses. For our first model, we combine the results of Sect. (2.3) with a parametrization of the four-quark condensates and the continuum.

3.1 Model I: Parametrizing the Condensates and Continuum

We first look at a simplified model in which we use the meson mass QCD sum rules in the $v_\parallel$, av, s, a, at, and t channels, assuming that the temperature dependence of the four-quark condensates behaves according to

$$\hat{Q}^\Gamma(T) = \hat{Q}^\Gamma(0)F(T). \tag{44}$$

For this simplified model, we will be assuming that the *same* function $F(T)$ describes the temperature dependence of all four-quark condensates. This amounts to a factorization assumption. For the present calculation, we take

$$F(T) = 1 - T^2/T_c^2, \tag{45}$$

which corresponds to the mean-field assumption of Ref. [7], where the temperature dependence of the quark condensate was taken to be proportional to the squar-root of this quantity. We take $T_c = 220$ MeV [5]. In our second model, discussed in Sect. 3.2, we will be able to check factorization for $T \neq T_c$.

Our second assumption regards the behavior of the temperature dependence of the continuum, which we are taking to begin at $s = S_0^\Gamma(T)$. As we are unable to calculate the temperature dependence of the continuum from first principles, we parametrize this quantity. We have found that we get reasonable and consistent solutions of our sum rules using all our models if we take

$$S_0^\Gamma(T) = S_0^\Gamma(0)G(T). \tag{46}$$

with

$$G(T) = 1 - T^2/\Lambda^2, \quad \text{with } \Lambda = 331\ MeV, \tag{47}$$

with $S_0(0)$ given in Table 3. As a result of this parametrization, our results depend on the value of the cutoff parameter Λ. With this choice of $S_0^\Gamma(T)$, the continuum moves down toward the resonance Γ as the temperature increases. Such behavior is expected to occur in a realistic system, to the extent that all the masses drop and hadronic resonances get increasing widths with increasing temperature.

Using these two assumptions, and the QCD sum rule specified by Eq. (31), we get equations for $m_\Gamma^2(T)$ of the following form,

$$m_\Gamma^2(T) = H^\Gamma(T, S_0^\Gamma(T), M_B^\Gamma, \hat{\hat{Q}}^\Gamma(T)). \tag{48}$$

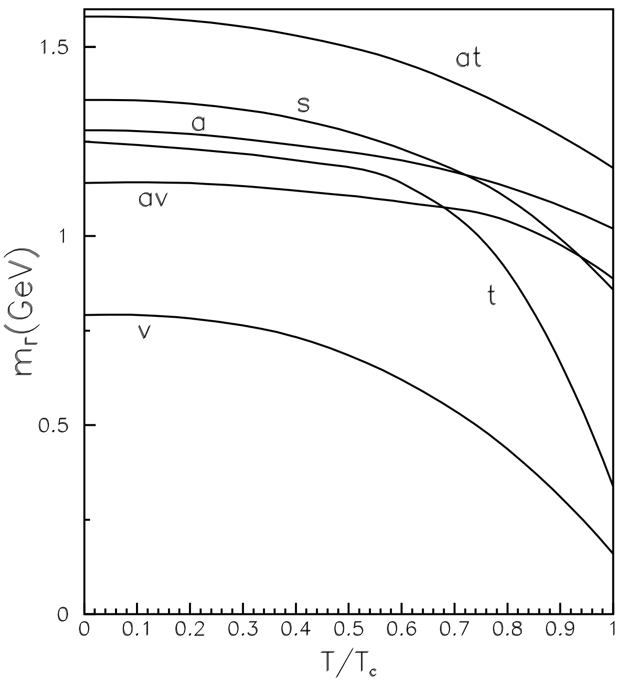

Figure 3: Solutions of Eq. (48) assuming factorized forms for four-quark condensates.

The L.H.S. of the correlators we use are given in Table 2. Note that in this model, $\hat{\hat{Q}}^{\Gamma}$ is proportional to $F(T)$ with the values at $T = 0$ given in Table 3. The solution of these equations is taken in the region where m_Γ is insensitive to M_B^Γ. We have found that to a reasonable approximation for the av, s, and a cases this region occurs where M_B^Γ attains its $T = 0$ value. However, for the $v_\parallel$, at, and t cases, the region of stability in M_B^Γ drops with increasing T. A simple, approximate description obtains by taking $M_B^{v_\parallel} \approx m_{v_\parallel}(T) + .5\ \mathrm{GeV}^2$, $M_B^{at} \approx m_{at}(T)$, and $M_B^t \approx m_t(T)$.

The results are shown in Fig. 3. As in Ref. [5], the mass of the vector meson drops to zero at $T \approx T_c$, but the rate of decrease is not nearly as fast here. The behavior of the tensor meson is particularly interesting. Between $T \approx 3T_c/5$ and $T \approx T_c$, $m_t^2(T, M_B^t)$ changes its character several times: from a function whose relevant region of stability is an asymptote at large M_B^t, to a function whose region of stability is a broad shoulder, and finally to a function having as its only stable solution one at relatively small values

of M_B^t. We are able to express the complicated change in the location of stable regions by $M_B^t \approx m_t(T)$. The axial tensor meson is also an interesting case. The relevant region of M_B^{at} is a broad shoulder at all T. The region of stability tends to move to smaller M_B^{at}, corresponding to smaller m_{as}, as T increases toward $T = T_c$. Additionally, in this limit the correlator becomes increasingly sensitive to the condensates.

It is easily verified from Fig. 3 and Table 3 that the gap between the continuum and the meson pole decreases as expected when T approaches T_c for all mesons except the tensor meson. In this case, the drop of the meson mass is more rapid than the dropping of $S_0(T)$. Note that the Borel transform becomes less effective at suppressing features of the excited states that are not modelled in the theory as the separation between the resonance and the continuum decreases. Thus, in this model as well as the next, the theoretical uncertainties grow as T_c is approached in a fashion that is increasingly dependent on the unknown parameter Λ in Eq. (47). These uncertainties may be able to be decreased somewhat by introducing specific forms of $F^\Gamma(\alpha)$ and $P_\alpha^\Gamma(\vec{p})$ given in Eq. (33) or by extrapolating to results obtained at asymptotically large T, where uncertainties may be again better controlled [23].

Although there is considerable theoretical uncertainty in our results in the vacinity of $T = T_c$, we note the following interesting feature that may hint at new physics. Starting at $T \approx T_c/2$ the tensor meson mass begins to drop quickly, and the rapid drop continues as T approaches T_c. Except for this case, and that of the vector meson, the meson masses remain relatively large at the transition temperature. The rapid decrease of the tensor meson mass appears to be associated with the unusual behavior of the corresponding correlator at large T, noted above. Because of this, the tensor meson may be particularly sensitive to the temperature dependence of the four-quark condensate and be an important case deserving careful experimental study.

3.2 Model II: Solving the Four-Quark Condensate Sum Rules

In this section, we remove one of the assumptions of the previous subsection, namely, we no longer parametrize the four-quark condensates but rather determine these condensates self-consistently using the four-quark QCD sum rules whose R.H.S. and L.H.S. are given respectively in Tables 7 and 8. Note that these four-quark sum rules depend on the masses of the mesons in the v, av, s, ps, and t_σ channels. In deriving these sum rules, we have already taken the meson mass on the R.H.S. to be the one corresponding to the lightest meson for each current. Thus, we take the mass $m_v(T)$ to be that corresponding to the $\rho-\omega$ meson; $m_{av}(T)$ to be that corresponding to the $a_1 - f_1$ meson; and $m_s(T)$ to be that corresponding to the $a_0 - f_0$ meson. Since J^{t_σ} couples to the 1^{--} channel, we take the mass in the t_σ channel to be $m_v(T)$, corresponding to the $\rho - \omega$ meson. For the mass of the pion, we take the free-space value $m_\pi = 140$ MeV following the observation in the model of Ref. [14] that the pion mass is rather stable with temperature up to T_c. The values of $m_s(T)$, $m_v(T)$, and $m_{av}(T)$ are of course taken to be solutions of the corresponding three meson-mass sum rules discussed already in Model I, above.

We may write the theory as follows. Note that the four-quark condensate sum rules may be solved algebraically for the five matrix elements $\langle\langle 0|J^\Gamma J^\Gamma|0\rangle\rangle$ if Eq. (41) is used to express $\hat{\hat{Q}}^\Gamma$ in terms of $\langle\langle 0|J^\Gamma J^\Gamma|0\rangle\rangle$. The result may be written

$$\langle\langle 0|J^\Gamma J^\Gamma|0\rangle\rangle(T) = K^\Gamma(T, m(T), S_0(T), M_B'^\Gamma), \tag{49}$$

i.e., as an explicit function of a Borel mass $M_B'^\Gamma$ for each matrix element $\langle\langle 0|J^\Gamma J^\Gamma|0\rangle\rangle$, the three relevant masses ($m(T)$), and the five threshold functions ($S_0(T)$). The remaining three equations are of course the three meson-mass sum rules of Eq. (48),

$$m_\Gamma^2(T) = H^\Gamma(T, m_\Gamma, S_0^\Gamma(T), M_B^\Gamma, \hat{\hat{Q}}^\Gamma(T)) \;\; \text{for } \Gamma = s,\; v,\; \text{and } av. \tag{50}$$

To get a completely self-consistent set of equations, we again express $\hat{\hat{Q}}^\Gamma$ in terms of $\langle\langle 0|J^\Gamma J^\Gamma|0\rangle\rangle$ using Eq. (41).

Our Model II thus consists of eight coupled nonlinear equations for the eight unknowns, consisting of the five matrix elements $\langle\langle 0|J^\Gamma J^\Gamma|0\rangle\rangle$ and the three meson masses $m_s(T)$, $m_v(T)$, and $m_{av}(T)$. As we are unable to calculate the temperature dependence of the continuum from first principles, we continue to parametrize this as in Eq. (46)

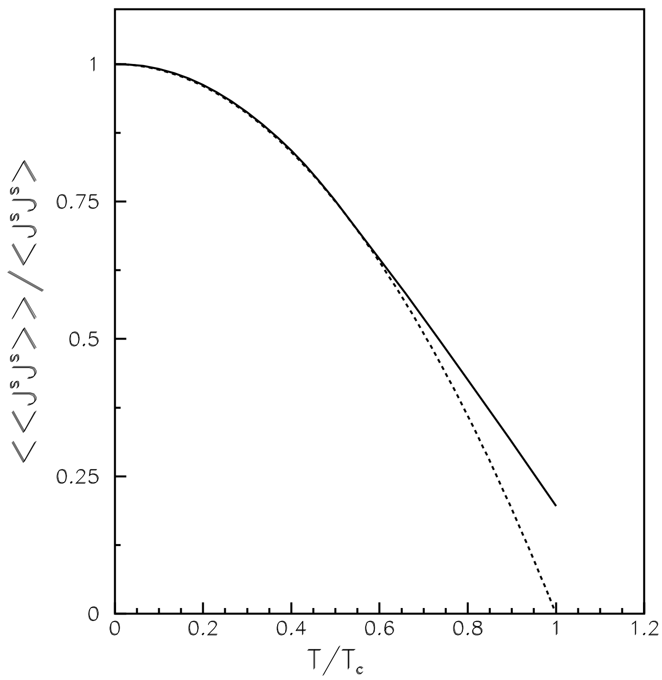

Figure 4: Temperature-dependence of $R(T)$ defined in Eq. (51) (solid line) for the scalar four-quark condensate compared to temperature dependence of four-quark condensates in the factorized, mean-field approximation (dashed line).

and Eq. (47), above. We take the thresholds S_0^Γ to be the same as those in the $T = 0$ meson-mass sum rules, Table 3, except in the case of the ps and t_σ channels, where we take $S_0^{t_\sigma} = S_0^v$ and $S_0^{ps} = 1300$ MeV. The parameters κ^Γ in Eq. (39) are obtained by solving the four-quark condensate sum rules at $T = 0$ and adjusting the values of these parameters to obtain the known *factorized* values of the matrix elements $\langle 0|J^\Gamma J^\Gamma|0\rangle$. Note that we are assuring that the quark condensates attain their free-space values by our choice of κ^Γ. The values that assure this are given in Table 9.

To obtain numerical solutions of the nonlinear system given in Eqs. (49, 50), we begin at $T = 0$, where we know that the solution of our equations gives the well-known results for the meson masses and (factorized) four-quark condensates. We then slowly increase the temperature, letting $T \to \delta$, solving by iteration the eight equations in

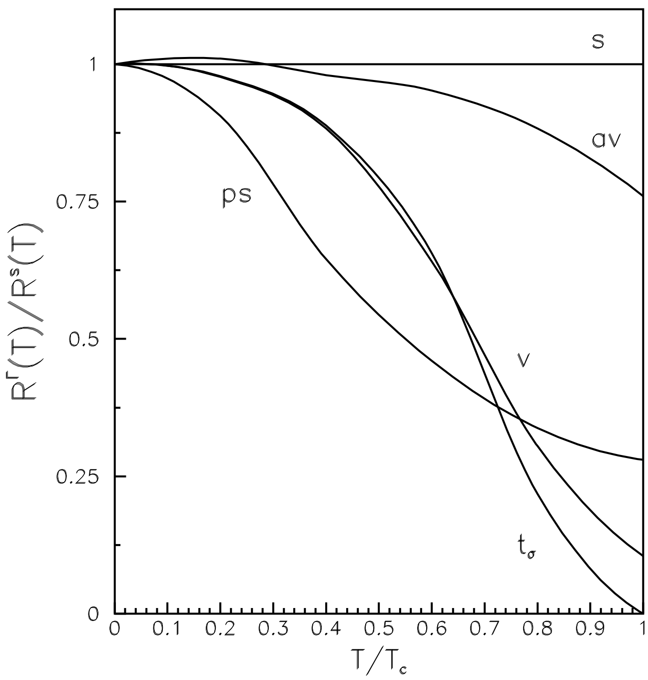

Figure 5: T-dependence of QCD sum rule determined four-quark condensates as a function of temperature. Note that the T-dependence is given relative to the the scalar four-quark condensate shown in Fig. 4.

eight unknowns until a stable solution is reached for the for the new meson masses and condensates at $T = \delta$. Then, we increase the temperature $T \to T + \delta$ and iterate again for a stable solution. The process is continued up to the transition temperature T_c. Our results do not rule out the existence of multiple solutions at finite temperature. We have in all cases presented the solutions that evolve continuously from the solutions at $T = 0$.

We obtain the solution of each sum rule by finding the region of stability as a function of the Borel mass for each quantity of interest. We thus find a Borel mass for each of the eight unknown quatities at each T. Once we know the condensates, we can solve the sum rules in the a, t, and at channels. Clearly, the meson masses in these three channels are determined as a consequence of the eight coupled equations, but the

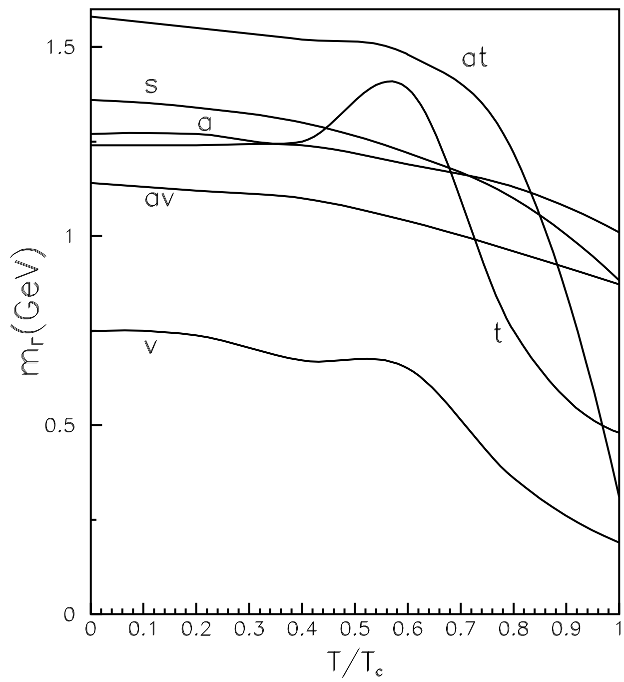

Figure 6: Meson masses as a function of temperature using our QCD sum rule determination of the four-quark condensates.

corresponding mesons do not appear as part of the four-quark condensates sum rule.

We give the numerical solutions for the coupled equations next. We will denote the the temperature dependence of the condensates by $R^\Gamma(T)$, so that

$$\langle\langle 0|J^\Gamma J^\Gamma|0\rangle\rangle = R^\Gamma(T)\langle 0|J^\Gamma J^\Gamma|0\rangle. \tag{51}$$

First, in Fig. 4, we show the behavior $R^\Gamma(T)$ for the four-quark scalar condensate. The solid curve shows the solution to our coupled equations, and the dashed line shows for comparison the mean-field temperature dependence given in Eq. (45). We see that the scalar four-quark condensate falls off with temperature very much as expected from factorized approximations [7]. We have found that the details of the temperature dependence in Fig. 4 is governed by our choice of $G(T)$ in Eq. (47).

Next, consider the relative temperature dependence of all of the four-quark con-

densates. These are shown in Fig. 5. We see here that the scalar condensate has the slowest fall-off with T. Additionally, it is clear that all condensates Γ fall off in T with different rates; the ratio $R^{\Gamma}(T)/R^{s}(T)$ is much less sensitive to $G(T)$ than $R^{\Gamma}(T)$ itself. Vacuum saturation would imply the same rate of fall-off, so we see that even at a modest temperature of $T \approx T_c/2$ there is a considerable violation of factorization.

At T=0 the factorization approximation of the four-quark condensates is also in question. In a study of the ratio of the isovector hadronic to the muon pair production in e^+e^- data [10] using a sum rule that weights the heavier states more than the usual sum rule [2], a value of the vector four-quark condensate is larger than the factorized value. Using this value we find that we get a satisfactory fit to the rho-meson mass. Moreover, in the present work, we observe that the $T = 0$, $B = 0$ sum rules give all meson masses consistent with experiment for our parameterization with $T = 0$ factorization. We have explored the additional features of four-quark condensates for finite T, and there is no difficulty for us to modify our results to include correction factors for $T = 0$.

We give the numerical solutions for the meson masses in Fig. 6. Note that the temperature dependence of the meson masses for all cases except the tensor and axial tensor mesons are quite similar to those of Model I shown in Fig. 3. Because the tensor and axial tensor show more sensitivity to the four-quark condensates, these cases may provide particularly interesting candidates to explore experimentally.

We have noted that accuracy of the correlator in the psudoscalar channel for the mass of the π meson is very poor. Novikov, et al. [24] have indicated that for channels that couple easily to the vacuum, such as the psudoscalar channel, instanton corrections identified as "direct instantons" may be particularly important. The subject of the instanton contributions to meson mass correlators has studied in some detail in a recent review [25]. Here different models of correlators have been compared, showing how instantons can improve the correlator out to large separations τ. Although direct instantons presumably also contribute to the psudoscalar and other four-quark condensate sum rules that we have used, we have normalized these sum rules to the factorized value of the four-quark condensate at $T = 0$. The fact that we are only relying on

the four-quark sum rules to provide the temperature dependence of the condensates and not their $T = 0$ values presumably improves the reliability of the sum rule for our purpose.

4 Summary and Conclusions

This paper has focused on the physics of the four-quark condensates $\hat{Q}^{\Gamma}$, which constitute fundamental, nonperturbative features of quantum chromodynamics needed for understanding how hadron masses (especially those of the light-quark mesons), as well as interactions among hadrons, arise. These as well as the other condensates all govern essential details of the phase transition that is believed to have occured as the early universe expanded and cooled from the era dominated by the quark-gluon plasma into the next, in which the quarks and gluons had coalesced into the familiar hadrons. Understanding the physics of this phase transition has been identified as one of the opportunities that presents itself to nuclear and particle physics community at the next generation of experimental facilities of RHIC and perhaps the LHC.

In spite of the importance of $\hat{Q}^{\Gamma} \equiv \langle\langle 0|\mathrm{\bar{q}\Gamma t^a q\bar{q}\Gamma t^a q}|0\rangle\rangle$, these quantities are currently quite poorly understood at both the phenomenological and at the theoretical levels. For example in the literature, the temperature dependence of $\hat{Q}^{\Gamma}$ is almost universally described through a factorized, mean-field approximation in which $\hat{Q}^{\Gamma} \propto \langle\langle\mathrm{\bar{q}q}\rangle\rangle^2 \propto 1 - \mathrm{T^2/T_c^2}$, with $T_c \approx 200$ MeV. In such a formulation, the dominant nonperturbative quark contribution to the masses of light-quark mesons is the quark condensate $\langle\langle\mathrm{\bar{q}q}\rangle\rangle$. In this paper, we have proposed a dynamical theory for the temperature dependence of $\hat{Q}^{\Gamma}$ that generalizes the familiar gap equation for the quark condensate $\langle\langle\mathrm{\bar{q}q}\rangle\rangle$. In our theory, the masses of the light-quark mesons are practically independent of $\langle\langle\mathrm{\bar{q}q}\rangle\rangle$; instead these masses depend on four-quark condensates that do not factorize for $T \neq 0$.

Our work is based on finite-temperature QCD sum rules in the Matsubara formalism and requires a model of the hadronic spectrum. The model of the hadronic spectrum we have chosen is the discretized model adopted in Ref. [5]. It contains, in addition, a

parameter that characterizes the temperature-dependence of the continuum. Equations are obtained coupling light-quark meson masses m_Γ and $\langle\langle 0|\bar{\mathrm{q}}\Gamma\mathrm{t^a q}\bar{\mathrm{q}}\Gamma\mathrm{t^a q}|0\rangle\rangle$. Stable solutions are found in which $\langle\langle 0|\bar{\mathrm{q}}\Gamma\mathrm{t^a q}\bar{\mathrm{q}}\Gamma\mathrm{t^a q}|0\rangle\rangle$ all decrease as T increases toward T_c, but at different rates, so that factorization becomes increasingly violated the farther T is varied away from $T = 0$.

We stress the connection between the temperature dependence of meson masses and the fundamental four-quark condensates in hot quark matter. The tensor 2^{++} mesons $\mathrm{a_2}(1320)$-$\mathrm{f_2}(1270)$ are particularly important for giving quantative information on the four-quark condensates. The temperature dependence of meson masses constitute predictions of our theory that may at least in principle be checked experimentally by measurement of final electromagnetic states ($\gamma\gamma$, e^+e^-, or $\mu^+\mu^-$) or perhaps semi-electromagnetic states such as $\pi\gamma$.

Our results do not depend on achieving thermodynamic equilibrium in relativistic heavy ion collisions, but only local thermodynamic equilibrium. We also note that the temperature- (and more generally, density-) dependence of meson masses may affect predictions of transport models modelling the evolution of matter into and out of the quark-gluon plasma. Results such as the ones we give may be incorporated in such models able to follow the evolution of the local temperature.

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Table 1: Meson mass correlator $\Pi'^{\Gamma}_{L.H.S.}(Q^2, T)$. Coefficients correspond to Eq. (19) for the av, s, and $v_{\parallel}$ cases, and to Eq. (23) for the a, t, and at cases. For the s case, there is also a glueball contribution, given in the text.

Case Γ	n	a_Γ	$n(\Gamma, b)$	b_Γ	$n(\Gamma, c)$	c_Γ
av	1	$\frac{1}{4\pi}\left(1 + \frac{\alpha_s}{\pi}\right)$	1	$-\frac{\pi}{12}$	2	$2\pi^2\alpha_s$
s	1	$\frac{3}{8\pi}\left(1 + \frac{11}{3}\frac{\alpha_s}{\pi}\right)$	1	$\frac{\pi}{8}$	2	$\pi^2\alpha_s$
$v_{\parallel}$	0	$\frac{1}{4\pi}\left(1 + \frac{\alpha_s}{\pi}\right)$	2	$\frac{\pi}{12}$	3	$-2\pi^2\alpha_s$
a	1	$\frac{1}{8\pi}\left(1 + \frac{\alpha_s}{\pi}\right)$	1	$-\frac{\pi}{24}$	2	$4\pi^2\alpha_s$
t	2	$\frac{3}{5\pi}\left(1 - \frac{\alpha_s}{\pi}\right)$	—	$-\frac{8\pi}{9}$	1	$-4\pi^2\alpha_s$
at	2	$\frac{3}{5\pi}\left(1 - \frac{\alpha_s}{\pi}\right)$	—	$-\frac{8\pi}{9}$	1	$-4\pi^2\alpha_s$

Table 2: Borel transform of meson mass correlator $\Pi'^{\Gamma}_{L.H.S}(M_B^2, T)$. Coefficients correspond to Eq. (25) in all cases.

Case Γ	n	A_Γ	$n(\Gamma, B)$	B_Γ	$n(\Gamma, C)$	C_Γ
av	1	$\frac{1}{4\pi}\left(1+\frac{\alpha_s}{\pi}\right)$	0	$-\frac{\pi}{12}$	1	$2\pi^2\alpha_s$
s	1	$\frac{3}{8\pi}\left(1+\frac{11}{3}\frac{\alpha_s}{\pi}\right)$	0	$\frac{\pi}{8}$	1	$\pi^2\alpha_s$
$v_\parallel$	0	$\frac{1}{4\pi}\left(1+\frac{\alpha_s}{\pi}\right)$	1	$\frac{\pi}{12}$	2	$-\pi^2\alpha_s$
a	1	$\frac{1}{8\pi}\left(1+\frac{\alpha_s}{\pi}\right)$	0	$-\frac{\pi}{24}$	1	$4\pi^2\alpha_s$
t	2	$\frac{3}{5\pi}\left(1-\frac{\alpha_s}{\pi}\right)$	-1	$-\frac{8\pi}{9}$	0	$-4\pi^2\alpha_s$
at	2	$\frac{3}{5\pi}\left(1-\frac{\alpha_s}{\pi}\right)$	-1	$-\frac{8\pi}{9}$	0	$-4\pi^2\alpha_s$

Table 3: Results for meson-mass sum rules, $T = 0$. Factorized values of the four-quark condensates are also given.

Case Γ	S_0^Γ (MeV^2)	m_i (MeV)	$< \hat{\hat{Q}}^\Gamma >_{fact} / < \bar{q}q >^2$
av	1700	1140	$-\frac{176}{81}$
s	3000	1360	$-\frac{176}{27}$
v	1500	748	$\frac{112}{81}$
a	2740	1240	$-\frac{20}{81}$
t	2500	1270	$-\frac{16}{9}$
at	3500	1580	$\frac{16}{9}$

Table 4: Definition of projection operator $P^{\Gamma}(p)$ and four-quark condensate correlator $\Pi'^{\Gamma}(Q^2, T)$.

Case Γ	$P^{\Gamma}(p)$	Def. Π'^{Γ}
av	$\frac{1}{m_{av}^2}\left(p_\mu p_\nu - p^2 g_{\mu\nu}\right)$	$-\frac{\pi p^2 g^{\mu\nu}}{6}\Pi^{av}_{\mu\nu}$
s	1	$\pi\Pi^{s}$
t_σ	$\frac{1}{m_{t_\sigma}^2}(p_\mu p_\lambda g_{\nu\rho} + p_\nu p_\rho g_{\mu\lambda}$	$\frac{\pi P^{\mu\nu,\rho\lambda}}{12}\Pi^{t_\sigma}_{\mu\nu,\rho\lambda}$
	$- p_\mu p_\rho g_{\nu\lambda} - p_\nu p_\lambda g_{\mu\rho})$	
ps	-1	$\pi\Pi^{ps}$
$v_\perp$	$\frac{1}{m_v^2}\left(p_\mu p_\nu - p^2 g_{\mu\nu}\right)$	$-\frac{\pi g^{\mu\nu}}{3}\Pi^{v}_{\mu\nu}$

Table 5: Four-quark condensate correlator $\Pi'^{\Gamma}_{L.H.S.}(Q^2, T)$. Coefficients corresond to Eq. (19).

Case Γ	n	a_Γ	$n(\Gamma, b)$	b_Γ	$n(\Gamma, c)$	c_Γ
av	2	$\frac{1}{4\pi}\left(1 + \frac{\alpha_s}{\pi}\right)$	—	0	1	$-\pi^2\alpha_s$
s	1	$\frac{3}{8\pi}\left(1 + \frac{11}{3}\frac{\alpha_s}{\pi}\right)$	1	$\frac{\pi}{8}$	2	$\pi^2\alpha_s$
t_σ	1	$-\frac{1}{8\pi}$	1	$\frac{\pi}{3}$	2	$16\pi^2\alpha_s$
ps	1	$\frac{3}{8\pi}\left(1 + \frac{11}{3}\frac{\alpha_s}{\pi}\right)$	1	$\frac{\pi}{8}$	2	$\pi^2\alpha_s$
$v_\perp$	1	$\frac{1}{4\pi}\left(1 + \frac{\alpha_s}{\pi}\right)$	1	$-\frac{\pi}{12}$	2	$2\pi^2\alpha_s$

Table 6: Fierz rearranged four-quark condensates.

Case Γ	$\hat{\hat{Q}}^{\Gamma}$	C_v	C_{av}	C_s	C_{ps}	C_t
v	$\hat{Q}^{av} + \frac{2}{9}\lvert J_0\rvert^2$	$\frac{29}{27}$	$-\frac{5}{9}$	$\frac{14}{9}$	$\frac{14}{9}$	0
av	$\hat{Q}^{v} + \frac{2}{9}\lvert J_0\rvert^2$	$\frac{11}{27}$	$\frac{11}{9}$	$-\frac{22}{9}$	$-\frac{22}{9}$	0
s	$\hat{Q}^{t} + \frac{2}{3}\lvert J_0\rvert^2$	$\frac{2}{9}$	$\frac{2}{3}$	$-\frac{22}{3}$	$\frac{14}{3}$	$\frac{1}{3}$
t_σ	$\hat{Q}^{s} + \frac{1}{36}\lvert J_0\rvert^2$	$-\frac{53}{108}$	$\frac{19}{36}$	$-\frac{11}{9}$	$\frac{4}{9}$	$-\frac{1}{4}$
a	$\hat{Q}^{ps} - \frac{1}{9}\lvert J_0\rvert^2$	$\frac{25}{54}$	$-\frac{11}{18}$	$-\frac{5}{18}$	$\frac{25}{18}$	$-\frac{1}{4}$
ps	$\hat{Q}^{at} + \frac{2}{3}\lvert J_0\rvert^2$	$\frac{2}{9}$	$\frac{2}{3}$	$-\frac{22}{3}$	$\frac{14}{3}$	$\frac{1}{3}$
t	$\hat{Q}^{v}$	$\frac{1}{3}$	1	-2	-2	0
at	$\hat{Q}^{av}$	1	$\frac{1}{3}$	2	2	0

Table 7: Four-quark correlator $\Pi'^{\Gamma}_{R.H.S.}(M_B^2, T)$.

Case Γ	$\Pi'^{\Gamma}_{R.H.S.}(M_B^2, T)$
av	$\frac{\langle\langle 0\|J^{av}J^{av}\|0\rangle\rangle}{\kappa^{av}}\frac{m_{av}^2}{2}\pi\ \coth\left(\frac{m_{av}}{2\tau}\right)e^{-m_{av}^2/M_B^2}$
s	$\frac{\langle\langle 0\|J^{s}J^{s}\|0\rangle\rangle}{\kappa^{s}}\pi\ \coth\left(\frac{m_{s}}{2\tau}\right)e^{-m_{s}^2/M_B^2}$
t_σ	$-\frac{\langle\langle 0\|J^{t\sigma}J^{t\sigma}\|0\rangle\rangle}{\kappa^{t\sigma}}\pi\ \coth\left(\frac{m_{v}}{2\tau}\right)e^{-m_{v}^2/M_B^2}$
ps	$-\frac{\langle\langle 0\|J^{ps}J^{ps}\|0\rangle\rangle}{\kappa^{ps}}\pi\ \coth\left(\frac{m_{ps}}{2\tau}\right)e^{-m_{ps}^2/M_B^2}$
$v_\perp$	$-\frac{\langle\langle 0\|J^{v}J^{v}\|0\rangle\rangle}{\kappa^{v}}\pi\ \coth\left(\frac{m_{v}}{2\tau}\right)e^{-m_{v}^2/M_B^2}$

Table 8: Four-quark correlator $\Pi'^{\Gamma}_{L.H.S.}(M_B^2, T)$.

Case Γ	n	A_Γ	$n(\Gamma, B)$	B_Γ	$n(\Gamma, C)$	C_Γ
av	2	$\frac{1}{4\pi}\left(1+\frac{\alpha_s}{\pi}\right)$	—	0	0	$-\pi^2\alpha_s$
s	1	$\frac{3}{8\pi}\left(1+\frac{11}{3}\frac{\alpha_s}{\pi}\right)$	0	$\frac{\pi}{8}$	1	$\pi^2\alpha_s$
t_σ	1	$-\frac{1}{8\pi}$	0	$\frac{\pi}{3}$	1	$16\pi^2\alpha_s$
ps	1	$\frac{3}{8\pi}\left(1+\frac{11}{3}\frac{\alpha_s}{\pi}\right)$	0	$\frac{\pi}{8}$	1	$\pi^2\alpha_s$
$v_\perp$	1	$\frac{1}{4\pi}\left(1+\frac{\alpha_s}{\pi}\right)$	0	$-\frac{\pi}{12}$	1	$2\pi^2\alpha_s$

Table 9: Values of κ^Γ and factorzied values of $< 0|\bar{q}\Gamma q\bar{q}\Gamma q|0 >$.

Case Γ	κ^Γ (GeV^2)	$\langle 0\| : \overline{q}\Gamma q\, \overline{q}\Gamma q : \|0\rangle / \langle 0\| : \overline{q}q : \|0\rangle^2$
av	0.0021	$\frac{1}{3}$
s	0.00073	$\frac{11}{12}$
t_σ	0.028	-1
ps	-0.00034	$\frac{1}{12}$
$v_\perp$	0.0036	$-\frac{1}{3}$